Problem 1 **1 Point**

Let $f(x)=|x|$. Which of the following rules are valid for all possible pairs of numbers x and y?

a) $f(x+y)=f(x)+f(y)$;
b) $f(-2x)=-2f(x)$;
c) $f(x+y)\le f(x)+f(y)$;
d) none of the above.

Problem 2 **1 Point**

If $f(x)=\ln|x-1|$, find the domain of f.

a) $(-\infty;1)\cup(1;\infty)$ b) $(1;\infty)$ c) $(-\infty;\infty)$ d) $(-1;1)$

Problem 3 **1 Point**

Which of the following transforms the graph of $y=3^x$ into the graph of $y+8=3^{x-5}$?

a) shift down by 8 units, left by 5 units;
b) shift down by 8 units, right by 5 units;
c) shift up by 8 units, left by 5 units;
d) shift up by 8 units, right by 5 units.

Problem 4 **2 Points**

Let f be the function defined on the set of all real numbers by the formula

$$f(x)=\begin{cases}\dfrac{x^2-9}{x+3}, & if \quad x\neq -3,\\ 3a, & if \quad x=-3.\end{cases}$$

For what value of a, the function f will be continues?

a) -6 b) -2 c) -3 d) 2

Problem 5 **2 Points**

If a fair coin is tossed tree times in succession find, the probability of getting exactly two tails, given that the second toss is a head.

a) $\frac{1}{2}$ b) $\frac{1}{8}$ c) $\frac{1}{4}$ d) $\frac{1}{3}$

Problem 6 **2 Points**

If $f(x) = x^2 \ln x$, then $f'(2) =$

a) $\sqrt{e}$ b) $2\ln(4e)$ c) 2 d) $4\ln 2$

Problem 7 **2 Points**

Let $f(x) = \dfrac{ax+b}{cx-a}$, where $x \in \left(-\infty; \dfrac{a}{c}\right) \cup \left(\dfrac{a}{c}; \infty\right)$. Find an inverse function $f^{-1}(x)$.

a) $f^{-1}(x) = \dfrac{cx-a}{ax+b}$;

b) $f^{-1}(x) = \dfrac{bx+a}{ax+c}$;

c) $f^{-1}(x) = \dfrac{ax+b}{cx-a}$;

d) $f^{-1}(x) = x$.

Problem 8 **1 Point**

Find the equation of tangent line of the graph of $y=\sqrt{2x^2+1}$ at the point $(2;3)$.

a) $y=12x-21$ b) $y=\frac{4}{3}x+\frac{1}{3}$ c) $y=\frac{1}{4}x+\frac{5}{2}$ d) $y=3$

Problem 9 **2 Points**

What is the range of the function $f(x)=\ln\left(x^2+1\right)$ over the domain $-2<x<3$?

a) $(5;10)$ b) $(\ln 5;\ln 10)$ c) $(0;\ln 10)$ d) $[0;\ln 10)$

Problem 10 **2 Points**

The rational function $R(x) = \dfrac{x^3 - 4x^2 + x + 6}{x + 5}$ has a

a) vertical asymptote at $x = -5$ and no horizontal asymptotes;
b) vertical asymptote at $x = 5$ and no horizontal asymptotes;
c) vertical asymptote at $x = 5$ and horizontal asymptote at $y = 1$;
d) vertical asymptote at $x = -5$ and horizontal asymptote at $y = 0$.

Problem 11 **2 Points**

$$\lim_{x \to 16} \frac{\sqrt[4]{x} - 2}{\sqrt{x} - 4} =$$

a) $\dfrac{1}{4}$ b) 2 c) $\dfrac{1}{2}$ d) does not exist

Problem 12 **Point 1**

Graph of function $y=1-x^4$ in xoy Cartesian coordinate system is symmetric

a) about the ox-axis;
b) about the line $x=1$;
c) about the origin o;
d) about oy-axis.

Problem 13 **2 Points**

If A and B are $n\times n$ matrices which from the following statements is false?

a) $(A-B)\cdot(A+B)=A^2-B^2$ if and only if $AB=BA$;
b) If A is invertible and $AB=AC$ then $B=C$;
c) If A and B are invertible matrices then AB is invertible and $(AB)^{-1}=B^{-1}A^{-1}$;
d) If $AB=0$ then $A=0$ or $B=0$.

Problem 14 **2 Points**

An entrance exam test in math at ISET contains 20 multiple-choice problems. Each problem has four choices for the answer, but only one of them is correct. Suppose a student randomly guesses the answer to each problem. Find the probability that student gets exactly 18 correct answers.

a) $\frac{855}{2^{39}}$ b) 0.1 c) $\frac{9}{2^{40}}$ d) $\frac{9}{2^{39}}$

Problem 15 **2 Points**

Let x and y be uniformly distributed, independent random variable on [0, 1]. The probability that the distance between points (x, y) and $\left(\frac{1}{2}; \frac{1}{2}\right)$ is more than $\frac{1}{2}$ is

a) $1-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{1}{2}$ d) $\frac{1}{3}$

Problem 16 **3 Points**

Let f be the function defined on the set of all real numbers by the formula

$$f(x)=\begin{cases}\sqrt{2-x}, \text{ if } x<2\\ x^3+k(x+1), \text{ if } x\geq 2\end{cases}$$

For what value of k, the limit $\lim\limits_{x\to 2} f(x)$ exists?

a) 0 b) 1 c) $-\frac{8}{3}$ d) 2

Problem 17 **3 Points**

On the interval $[1;10]$, absolute maximum of the function $y=\frac{\ln^2 x}{x}$ is

a) 0;
b) 1;
c) $\frac{4}{e^2}$;
d) $\frac{(\ln 10)^2}{10}$.

Problem 18 **3 Points**

Let $f(x) = 2^x$. Using the chain rule, determine an expression for the derivative of $\left[f(g(x)) \right]$.

a) $2^{g'(x)} \ln 2$;
b) $2^{g(x)} \cdot g'(x)$;
c) $x \cdot 2^{g(x)-1} \cdot g'(x)$;
d) $2^{g(x)} \cdot g'(x) \cdot \ln 2$.

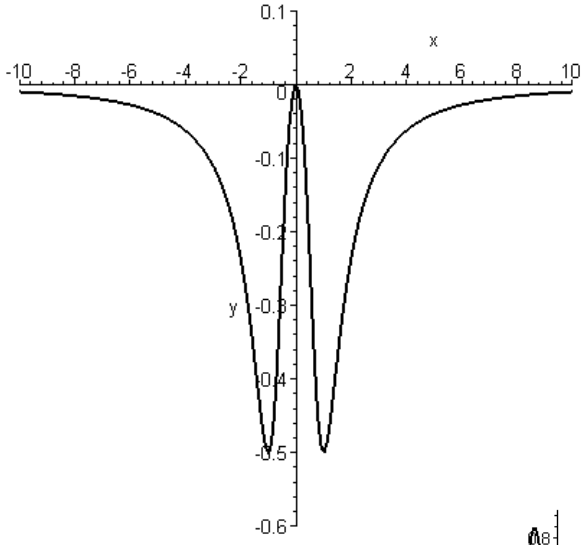
Problem 19 **3 Points**

If $f(x) = \frac{x^3}{3} + x^2 - 24x + 8$, then f is

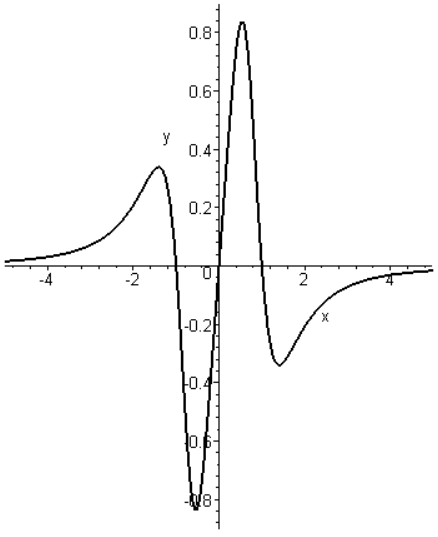
a) increasing on $(-6; 4)$ and has a relative minimum when $x = 4$;
b) decreasing on $(-\infty; -6)$ and has a relative maximum when $x = -6$;
c) increasing on $(4; \infty)$ and has a relative maximum when $x = 4$;
d) decreasing on $(-6; 4)$ and has a relative minimum when $x = 4$.

Problem 20 **3 Points**

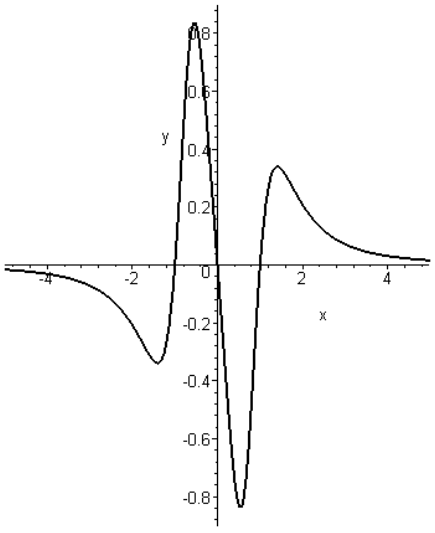
The graph of a function $y = f(x)$ is given. Which of the following could be the graph of $y = f'(x)$?



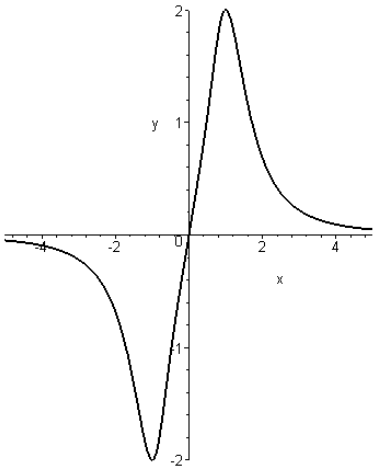
a)



b)



c)



d)