Problem 1 **1 Point**

If $f(x)=\sqrt{\dfrac{x+1}{|2x-1|}}$, find the domain of f.

a) $\left[-1;\dfrac{1}{2}\right)$

b) $\left[-1;\dfrac{1}{2}\right)\cup\left(\dfrac{1}{2};\infty\right)$

c) $(-\infty;-1]\cup\left(\dfrac{1}{2};\infty\right)$

d) $(-\infty;\infty)$

Problem 2 **1 Point**

If $f(x)=\begin{cases}\sqrt{1-x^2} & \text{if } -1\le x<0\\ |x-1| & \text{if } x>0\end{cases}$, find all solutions of the equation $f(x)=x$.

a) $\left\{\dfrac{1}{\sqrt{2}};-\dfrac{1}{\sqrt{2}}\right\}$;

b) $\left\{-\dfrac{1}{\sqrt{2}};\dfrac{1}{2}\right\}$;

c) $\left\{\dfrac{1}{2}\right\}$;

d) $\left\{\dfrac{1}{\sqrt{2}};-\dfrac{1}{\sqrt{2}};\dfrac{1}{2}\right\}$

Problem 3 **1 Point**

Which of the following transforms the graph of $y=\frac{1}{2^x}$ into the graph of $y+8=\frac{1}{2^{x-5}}$?

a) shift down by 8 unit, left by 5 unit;
b) shift down by 8 unit, right by 5 unit;
c) shift up by 8 unit, left by 5 unit;
d) shift up by 8 unit, right by 5 unit.

Problem 4 **2 Points**

Let f be the function defined on the set of all real numbers by the formula

$$f(x)=\begin{cases}\frac{x^3-8}{x-2}, & if \quad x\neq 2,\\ a+1, & if \quad x=2.\end{cases}$$

For what value of a the function f will be continuous?

a) 11 b) 10 c) 3 d) 12

Problem 5 **2 Points**

A fair die is rolled 4 times. What is the probability of getting exactly 2 fives?

a) $\frac{1}{36}$ b) $\frac{1}{3}$ c) $\frac{25}{6^4}$ d) $\frac{25}{216}$

Problem 6 **2 Points**

If $f(x) = x^2 \sin x$, then $f'(\pi) =$

a) $-\pi^2$ b) -2π c) 0 d) $\frac{\pi}{2}$

Problem 7 **2 Points**

Consider functions f and g such that $f(g(x)) = \sqrt{x^2+1} - 1$. If $g(x) = x^2 + 1$, then what is the value of $f(4)$?

a) 17
b) 3
c) 1
d) Data isn't sufficient to find $f(4)$

Problem 8 **1 Point**

Find the point at which the tangent line of the graph of function $y = x^4 + 1$ is parallel to the line $y = -32x + 1$.

a) $(-2; 17)$
b) $(2; 17)$
c) $(1; 2)$
d) Does not exist.

Problem 9 **2 Points**

A circle has $A(-2; -3)$ and $B(6; 1)$ as the endpoints of the diameter. Find the equation of this circle.

a) $2y - x + 4 = 0$
b) $(x-2)^2 + (y+5)^2 = 20$
c) $(x-1)^2 + (y-1)^2 = 25$
d) $(x-2)^2 + (y+1)^2 = 20$

Problem 10 **2 Points**

The rational function $R(x) = \dfrac{2x-6}{x+5}$ has a

a) vertical asymptote at $x = -5$ and no horizontal asymptotes;
b) vertical asymptote at $x = 5$ and no horizontal asymptotes;
c) vertical asymptote at $x = -5$ and horizontal asymptote at $y = 3$;
d) vertical asymptote at $x = -5$ and horizontal asymptote at $y = 2$.

Problem 11 **2 Points**

$\lim_{x\to 1}\frac{\sqrt{x}-1}{x^2-x}=$

a) 0 b) 1 c) ∞ d) $\frac{1}{2}$

Problem 12 **Point 1**

Find the inverse of matrix $A=\begin{pmatrix}3 & 4\\ 5 & 6\end{pmatrix}$.

a) $\begin{pmatrix}\frac{1}{3} & \frac{1}{4}\\ \frac{1}{5} & \frac{1}{6}\end{pmatrix}$;

b) $\begin{pmatrix}6 & -4\\ -5 & 3\end{pmatrix}$

c) $\begin{pmatrix}-3 & 2\\ \frac{5}{2} & -\frac{3}{2}\end{pmatrix}$

d) $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

Problem 13 **2 Points**

Find a function the graph of which is symmetric with respect to the origin of the *xoy* Cartesian coordinate system to the graph of function $f(x) = x^3 - 3x^2 + 2x + 4$.

a) $g(x) = -x^3 - 3x^2 - 2x + 4$;
b) $h(x) = x^3 + 3x^2 + 2x - 4$;
c) $k(x) = -x^3 + 3x^2 - 2x - 4$;
d) $u(x) = -x^3 + 3x^2 - 2x + 4$.

Problem 14 **2 Points**

If f^{-1} is an inverse function of the function f and $f'(x_0) \neq 0$ then

a) $\left(f^{-1}\right)'\left(f(x_0)\right) = \frac{1}{f'(x_0)}$;
b) $\left(f^{-1}\right)'(x_0) = \frac{1}{f'(x_0)}$;
c) $\left(f^{-1}\right)'\left(f^{-1}(x_0)\right) = \frac{1}{f'(x_0)}$;
d) $\left(f^{-1}\right)'(x_0) = f'(x_0)$.

Problem 15 **2 Points**

Let x and y be uniformly distributed, independent random variable on [0, 1]. The probability that the minimum between x and y is less than $\frac{1}{4}$ is

a) $\frac{1}{16}$ b) $\frac{7}{16}$ c) $\frac{1}{2}$ d) $\frac{5}{16}$

Problem 16 **3 Points**

Let f be the function defined on the set of all real numbers by the formula

$$f(x)=\begin{cases}\dfrac{e^x-1}{a\left(e^x-e^{2x}\right)}, & \text{if } x<0\\ a\sqrt{x+1}+2, & \text{if } x\geq 0\end{cases}.$$

For what value of a, the limit $\lim_{x\to 0} f(x)$ exists?

a) -2 b) 1 c) $\ln 2$ d) -1

Problem 17 **3 Points**

On the interval $[-1; 3]$, absolute minimum of the function $y = x^3(2x-1)^4$ is

a) 0;
b) 1;
c) -81;
d) -85.

Problem 18 **3 Points**

The sum of two nonnegative numbers, x and y, is 12. Find the largest possible value of $x^2 y$.

a) 256 b) 128 c) 144 d) 216

Problem 19 **3 Points**

If $f(x)=(x-1)(x+2)^2$, then f is

a) increasing on $(-\infty;-2)$ and on $(0;\infty)$ and has a relative minimum when $x=-2$;

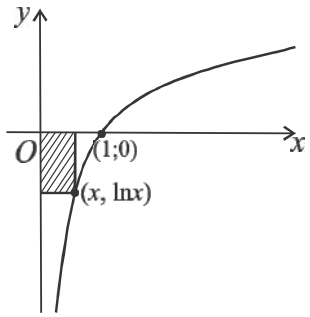
b) decreasing on $(-2;0)$ and has a relative maximum when $x=0$;

c) decreasing on $(-2;0)$ and has a relative maximum when $x=-2$;

d) increasing on $(-\infty;-2)$ and on $(0;\infty)$ and has a relative maximum when $x=0$.

Problem 20 **3 Points**

A rectangle in the fourth quadrant of the xy-plane has adjacent sides on the coordinate axes (see figure). If the vertex opposite the origin is on the curve $y=\ln x$, what is the maximum area this rectangle can have?



a) $\sqrt{e}$ b) 1 c) $\frac{1}{e}$ d) e