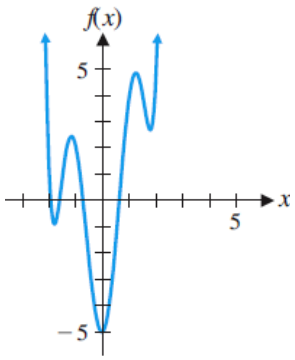
Problem 1 **1 Point**

What is the minimum degree of a polynomial function that could have the graph in the picture.



a) 4
b) 5
c) 6
d) 8

Problem 2 **1 Point**

f is a quadratic function with maximal value 5 at point $x=-3$. Find the sum of the roots of the equation $f(x)=0$.

a) -6;
b) 6;
c) -15;
d) -3

Problem 3 **1 Point**

Which of the following transforms the graph of $y = 3^{-x+2}$ into the graph of $y + 5 = \frac{1}{3^{x-5}}$?

a) shift down by 5 unit, right by 7 unit;
b) shift down by 5 unit, right by 3 unit;
c) shift up by 5 unit, left by 3 unit;
d) shift up by 5 unit, left by 7 unit.

Problem 4 **2 Points**

Let f be the function defined on the segment $[0; 6]$ by the formula

$$f(x) = \begin{cases} \dfrac{\sqrt{x} - \sqrt{6-x}}{x-3}, & if \quad x \neq 3, \\ k, & if \quad x = 3. \end{cases}$$

For what value of k, the function f will be continuous?

a) $\sqrt{3}$ b) 0 c) $\frac{1}{\sqrt{3}}$ d) 2

Problem 5 **2 Points**

A fair coin is tossed 6 times. What is the probability of getting the equal number of heads and tails?

a) $\frac{1}{8}$ b) $\frac{1}{2}$ c) $\frac{5}{16}$ d) $\frac{25}{64}$

Problem 6 **2 Points**

If $f(x) = e^x \ln\left(x^2+1\right)$, then $f'(1) =$

a) $e\ln 2$ b) $e\ln(2e)$ c) 0 d) e

Problem 7 **2 Points**

A function $f:(0;+\infty)\to R$ is defined by the formula $f(x)=\sqrt{x+\frac{1}{x}}$. Find the range of f.

a) $[0;+\infty)$

b) $(0;+\infty)$

c) $(-\infty;+\infty)$

d) $\left[\sqrt{2};+\infty\right)$

Problem 8 **1 Point**

Find a point at which the tangent line of the graph of function $y=\frac{1}{4}x^4-3x+1$ is orthogonal to the line $y=-0.2x+1$.

a) $(-2;11)$

b) $(2;-1)$

c) $\left(1;-\frac{7}{4}\right)$

d) Does not exist.

Problem 9 **2 Points**

Find the equation of the circle which is symmetric to the circle $(x-2)^2+(y+2)^2=8$ with respect to the line $y=-x$.

a) $(x-2)^2+(y-2)^2=8$
b) $(x+2)^2+(y+2)^2=8$
c) $(x+2)^2+(y-2)^2=8$
d) $(x-2)^2+(y+2)^2=8$

Problem 10 **2 Points**

The rational function $R(x)=\dfrac{2x-10}{x^2-25}$ has a

a) vertical asymptote at $x=-5$ and horizontal asymptote $y=0$;
b) vertical asymptotes at $x=5$ and $x=-5$ and horizontal asymptote $y=0$;
c) vertical asymptotes at $x=5$ and $x=-5$ and no horizontal asymptote;
d) vertical asymptote at $x=-5$ and no horizontal asymptote.

Problem 11 **2 Points**

$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{\frac{n}{4}} =$

a) $\sqrt{e}$ b) $2e$ c) $\frac{e}{2}$ d) e^2

Problem 12 **Point 1**

Find the sum of all elements of the matrix $\begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix}^{-1}$.

a) $-\frac{2}{11}$;

b) $\frac{6}{11}$

c) $-\frac{6}{11}$

d) $\frac{2}{11}$

Problem 13 **2 Points**

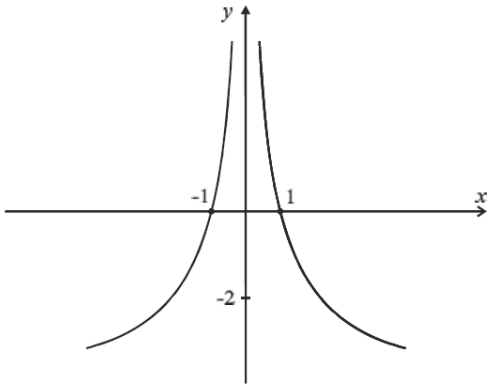
Find a function the graph of which is given on the picture.

a) $f(x)=\log_{0.5}|x|$;

b) $h(x)=\dfrac{1}{0.5|x|}-2$;

c) $k(x)=0.5^{|x|}$;

d) $g(x)=\dfrac{1}{\log_{0.5}|x-1|}$.



Problem 14 **2 Points**

Find $p+q$ if it is known that the equation of the tangent line of the graph of function $y=x^2+px+q$ at point $x=1$ is $y=3x-2$.

a) -1;
b) 3;
c) 1;
d) 0.

Problem 15 **2 Points**

Find the interval(s) on which the function $y = 1 + \frac{1}{x} + \frac{1}{x^2}$ is increasing.

a) $(-\infty; -2)$ and $(0; \infty)$;
b) $(-\infty; 0)$;
c) $(-2; 0)$;
d) $(1; \infty)$.

Problem 16 **3 Points**

Find the value of a parameter c if it is known that the function $f(x) = xe^{-cx}$ takes the maximal value at the point $x = 3$.

a) 3 b) $\frac{1}{3}$ c) 0 d) 1

Problem 17 **3 Points**

Let x and y be uniformly distributed, independent random variables on [0, 1]. The probability that the maximum between x and y is less than $\frac{1}{2}$ and more than $\frac{1}{3}$ is

a) $\frac{2}{9}$ b) $\frac{1}{6}$ c) $\frac{1}{3}$ d) $\frac{5}{36}$

Problem 18 **3 Points**

Find maximum value for the function $f(x)=\frac{x^3}{e^x}$ on the interval $(-\infty;+\infty)$.

a) $\frac{1}{e}$ b) 2 c) $\frac{27}{e^3}$ d) Does not exist

Problem 19 **3 Points**

A function f is defined by the formula $f(x)=\begin{cases} px^3+3x+q, \text{ if } x\le 1 \\ 2x+7, \text{ if } x>1 \end{cases}$. Find the values of parameters p and q for which $f(x)$ is differentiable function.

a) $p=-\frac{1}{3};\ q=\frac{19}{3}$

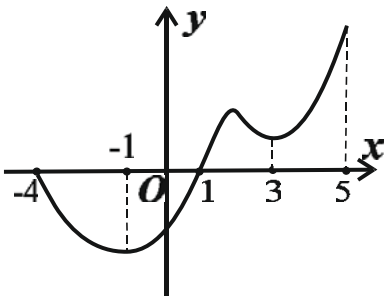
b) $p=1;\ q=5$

c) $p=-1;\ q=7$

d) $p=-\frac{2}{3};\ q=\frac{20}{3}$

Problem 20 **3 Points**

The differentiable function $y=f(x)$ is defined on the segment $[-4;\ 5]$. On the picture the graph of a derivative function $y=f'(x)$ is given. Find the point where the function $y=f(x)$ achieves the absolute minimum.



a) -1 b) 1 c) 3 d) 0